Low Pm turbulence in Protoplanetary discs

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Protoplanetary disks

- **Size:** $10^{11} - 10^{15}$ cm
- **Temperature:** $10^3 - 10^1$ K
- **Number density:** $10^{10} - 10^{17}$ cm$^{-3}$
- **Ionization fraction:** $\sim 10^{-13}$
Magnetorotational instability

Main properties

- Due to an interaction between magnetic tension and epicyclic motions
- Not too strong magnetic fields required («weak field instability»)
- Need a sufficiently high ionization fraction

Balbus & Hawley 1991, Balbus 2003
Protoplanetary disks

Courtesy of M. Flock
The incompressible shearing box model

Vertical and toroidal total magnetic flux conserved

Mean vertical field
("guide field")

Zero mean field

Boundary conditions

- Use shear-periodic boundary conditions = «shearing-sheet»
- Periodic in \( y \) and \( z \) (non stratified box)

Radial turbulent transport

\[
\alpha = \frac{\langle \delta v_x \delta v_y - \delta B_x \delta B_y \rangle}{\Omega^2 H^2}
\]
MRI simulations

Typical simulation

Orbits: 5.973616
Protoplanetary discs $Pm \sim 10^{-5}$

Reaction of small scales ($Pm$) on large scales ($\alpha$)

MHD subgrid model should be a function of $Pm$?

$Pm$ vs large scale transport with a guide field

Longaretti & Lesur 2010

Longaretti & Lesur 2010

$20000 < Re < 60000$

$\beta = 100$

$\beta = 1000$

$\beta = 10000$

$1/16$ $1/8$ $1/4$ $1$ $4$ $8$ $Pm$

$10000$ $0.045$

$0.03$

$0.045$

$0.005$ $0.03$ $0.04$
Pm vs large scale transport without a guide field

No turbulence for Pm<2?

Fromang et al. 2007
ILES and no guide field MRI: a worst case scenario?

- 64x100x64
- 128x200x128
- 256x400x256

Fromang & Papaloizou 2007

ILES fails to converge to a solution

Not true anymore for stratified boxes (Davis et al. 2010)
Introducing LES for the sub-resistive cascade (à la Ponty et al. 2004)

\[
\begin{align*}
E_{\text{Mag}}(k) \\
E_{\text{Kin}}(k)
\end{align*}
\]

Turbulent cascade

\[
\begin{align*}
1/H & \quad \text{Energy injection} \\
1/l_\eta & \quad \text{Pm<1}
\end{align*}
\]

Turbulent transport

Chollet Lesieur 1981
Reference DNS

Re=60000
Rm=600
Pm=0.01
384 pts/H
Anisotropy

- No anisotropy associated to the guide field ($\frac{\delta B}{B_0^z} \gg 1$)
- Strong x-y anisotropy due to the shear
- Chollet-Lesieur not adapted in this case...
LES vs DNS spectra

Kinetic energy

Magnetic energy

Computation time gain $\sim 100$

Have we reached an asymptotic regime for $Pm \ll 1$?
LES vs DNS flows

DNS 384 pts/H

LES 96 pts/H
Conclusions

• MRI-driven turbulence is sensitive to small scale processes.

• Should MHD LES models depend on small scale physics? (locality ?)

• ILES sometimes fail to converge. In general, it automatically implies Pm~2-3.

• In the Pm<<1 limit, Chollet-Lesieur works well for the hydro cascade, despite the strong anisotropy (ILES for hydro seems to work as well)

LES challenges in the MRI context:

• MRI without a guide field in the Pm<1 regime.

• Asymptotic regime in the Pm>>1 limit (full LES-MHD model)