Summary.

1. Kinetic MHD models of weakly collisional astrophysical plasmas require a SGS model that captures the effect of microscopic instabilities (firehose and mirror) on pressure anisotropy, heat fluxes, and the Hall term at high $\beta$.

2. Local hybrid PIC studies can be used to study saturation of these instabilities, and to develop SGS models.

3. Local hybrid PIC simulations can be used to validate these SGS models in kinetic MHD studies of the MRI.
Kinetic Effects in Astrophysical MHD

**Example:** X-ray emitting plasma in clusters of galaxies is in the *kinetic MHD* regime: $\lambda << L, \lambda >> \rho$ (gyro-radius)

For example: Abell 1689

Purple=Chandra X-ray image
Yellow=Hubble optical image

$T \sim 4.5 \text{ keV}, n \sim 10^{-3}-10^{-4} \text{ cm}^{-3}, B \sim 1\mu\text{G}$ implies $\lambda_{\text{mfp}} \sim 10^{22} \text{ cm} \sim 0.1R_V$

$\rho \sim 10^8 \text{ cm}$

Global PIC simulations impossible
Simplest model of low-collisionality regime, use continuum (fluid) equations plus anisotropic transport coefficients (Braginskii 1965):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \mathbf{v}] = 0
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + P^*] = -\nabla \cdot \Pi
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*) \mathbf{v} - \mathbf{B} (\mathbf{B} \cdot \mathbf{v})] = -\nabla \cdot \mathbf{Q} - \nabla \cdot [\Pi \cdot \mathbf{v}]
\]

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0
\]

\[
E = \frac{P}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2}, \quad \text{Definition of total energy}
\]

\[
\mathbf{Q} = -\chi \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla T \quad \text{Anisotropic heat flux (\(\chi = \text{conductivity}\))}
\]

\[
\Pi = -3\eta \left( \hat{\mathbf{b}} \hat{\mathbf{b}} - \frac{1}{3} \mathbf{l} \right) \left( \hat{\mathbf{b}} \hat{\mathbf{b}} - \frac{1}{3} \mathbf{l} \right) : \nabla \mathbf{v} \quad \text{Anisotropic viscous stress tensor}
\]
New dynamics in kinetic MHD: magneto-thermal instability

Balbus 2000; Parrish & Stone 2005; 2007

With anisotropic conduction, atmospheres with temperature decreasing upward are convectively unstable, regardless of entropy profile.

Kunz et al. 2012

Colors = Temperature
Lines = B-field
Problems with Braginskii

Large pressure anisotropies in the plasma drive instabilities at microscopic (close to Larmor radius) scale.

When $P_{\text{perp}} \ll P_{\text{para}}$: firehose instability

$P_{\text{perp}} \gg P_{\text{para}}$: mirror instability

But Braginskii gets the wrong growth rates for both. Moreover, fastest growth rate is near Larmor radius, which simulations can never resolve.

- Saturation of firehose and mirror at small scales can strongly affect MHD on large scales by tangling field and limiting P anisotropy

- Need sub-grid model for firehose and mirror at large $\beta$
Developing a sub-grid model with hybrid PIC.

Study **firehose instability** in decreasing magnetic field.

\[ B_x = \text{constant} \]
\[ |B_y| \text{ decreasing} \]

From adiabatic invariance of
\[ \mu \propto m v^2 / B \sim T / B, \text{ must have } v \text{ decreasing} \]

**mirror instability** in shear amplification of magnetic field.

\[ B_x = \text{constant} \]
\[ |B_y| \text{ increasing} \]

From adiabatic invariance of
\[ \mu \propto m v^2 / B \sim T / B, \text{ must have } v \text{ increasing} \]
Firehose and mirror with hybrid PIC

(M. Kunz & JMS)

Fluid electrons and kinetic ions
Written entirely new code using novel algorithms for some steps
Uses $\delta f$ methods to achieve extremely low noise.

**Test:** dispersion relation for whistler waves

**Early results:** driven firehose saturates by tangling field
Kinetic MRI studied with hybrid PIC  
(M. Kunz & JMS)

- Local shearing-box simulation of the MRI
- Box size $2120 \times 2120 \times 1060$ c/$\omega_{p,I}$
- Roughly 2 billion particles, $256^3$ grid

- Box is big enough to be in MHD regime on large scales, while still capturing kinetic effects on scales of Larmor radius.
- Direct comparison to kinetic MHD simulations can be used to validate SGS model.
Summary.

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New hybrid-PIC code

Lagrangian ion particles solve 6D characteristics of

\[
\left( \frac{\partial}{\partial t} - \sigma_0 x \frac{\partial}{\partial y} \right) f_s + v' \cdot \nabla f_s \]
\[
+ \left[ \frac{q_s}{m_s} \left( E' + \frac{v'}{c} \times B \right) - 2\omega_0 \hat{e}_z \times v' + \sigma_0 v'_x \hat{e}_y + g_{\text{eff}} \right] \cdot \frac{\partial f_s}{\partial v'} = 0
\]

using a generalized Boris push for shearing box with

\[ v' \equiv v + \sigma_0 x \hat{y} \] (orbit preserving)

Electrons are massless fluid; provide Ohm’s law:

\[
E + \frac{u_i}{c} \times B = -\frac{T_e \nabla n_i}{e n_i} + \frac{(\nabla \times B) \times B}{4\pi Z e n_i}
\]

2nd-order accurate constrained transport solution to

\[
\left( \frac{\partial}{\partial t} - \sigma_0 x \frac{\partial}{\partial y} \right) B = -c \nabla \times E' - \sigma_0 B_x \hat{e}_y
\]

with upwinding along characteristics and optional orbital advection (for SB)