3D turbulence resolving simulation on convective sedimentation of fine sediment in the coastal environment

XIAO YU, TIAN-JIAN HSU AND S. BALACHANDAR
Introduction – sediment transport

• **Initial Deposition process**
  - Sediments start to settle when river enters the ocean.

• **Re-suspension process**
  - Both terrestrial and marine origin sediment can be re-suspended and redistributed by the combined motion of wave and current.

Fate of fine sediment (adopted from Farnsworth and Warrick USGS 2008)

- **Primary particle** ~ 0.1mm/s
- **Floc** ~ 1mm/s
- **Convective sedimentation** ~ cm/s
Initial deposition process - Sedimentation in stratified ambient flow

- Two-layer system with denser salt water in the lower layer and lighter sediment-water mixture in the upper layer.
- Initially the flow is at rest and stably stratified.
- The sediment particle is fine, in this study in the range of 2μm to 60 μm.
- The salt concentration is fixed, with salt water density 1027 kg/m³.
- Interfacial instabilities may occur
  - Settling-driven mechanism (R-T instability)
  - Double-diffusive mechanism

Adopted from Hoyal et. al 1999 JGR
Equilibrium Eulerian Approach

**Assumptions**
- Fine sediment particles grain size $< 100 \mu m$ ($St << 1$).
- Dilute flow, inter-granular collisions are neglected.
- Boussinesq approximation
- Equilibrium approximation

**Governing Equations**

\[
\frac{\partial \vec{u}}{\partial t} + \nabla \cdot (\vec{u} \vec{u}) = -\nabla p + \nabla^2 \vec{u} - (Ra\phi + Ra_s\phi^s)\vec{e}_3
\]

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{v}) = \frac{1}{Sc} \nabla^2 \phi
\]

\[
\frac{\partial \phi^s}{\partial t} + \nabla \cdot (\phi^s \vec{u}) = \frac{1}{Pr} \nabla^2 \phi^s
\]

\[
Pr = \frac{\nu}{k_s}, \quad Sc = \frac{\nu}{\kappa}, \quad Ra_s = \frac{\beta g L^3 \Delta \Phi_s}{\nu^2}
\]

\[
Ra = \frac{\alpha g L^3 \Delta \Phi}{\nu^2}, \quad V_0 = \frac{V}{U}
\]

\[
\phi = (1 + \text{erf}((z - \varepsilon) / \delta_p)) / 2
\]

\[
\phi^s = (1 + \text{erf}(z / \delta_s)) / 2
\]
Sediment Diffusivity

- For very fine sediment particles, the Brownian motion dominates the diffusion process of particles (Einstein Relation).
- For coarser sediment particles (for our study, $d = 2\sim60 \mu m$), the long-range hydrodynamic effect dominates, and Brownian motion can be neglected.

$$
\kappa(\phi) \approx \frac{\Delta V \xi}{2.5} \quad \text{with} \quad \frac{\Delta V}{V} = \sqrt{11} \phi^{1/3} \sqrt{S(\phi,0)} \\
\xi = 5.5d\phi^{-1/3}
$$

For dilute flow $S(\phi,0) \approx 1.0$  \hspace{1cm} \kappa = 0.2 \sqrt{(11)^3} Vd \sqrt{S(\phi,0)}$
Linear Stability Analysis

- Linear stability analysis is carried out to provide some insight on the key parameters which control the dynamics of the system.
  - The ratio of two length scales, namely the settling distance and the diffusion length $\frac{\varepsilon}{\delta_p}$.
  - The ratio of density differences induced by salt and sediment $\Lambda = \frac{\beta \Delta \Phi}{\alpha \Delta \Phi^s}$.

\[
\frac{Z}{H^*} = \frac{\varepsilon}{\delta_p} = \frac{\varepsilon}{\delta_s}
\]

\[
\frac{R}{G^*} (s^{-1})
\]

\[
\text{Growth rate} \ G (s^{-1})
\]

\[
Wavenumber \ (\text{cm}^{-1})
\]

\[
\text{D=2µm} \quad \text{D=20µm}
\]

\[
\varepsilon = 0
\]

\[
\delta_p = \delta_s
\]
Linear stability analysis

D = 2µm

\[ \varepsilon = V_0 t \]

\[ \delta_p = \sqrt{4Sc(t + t_0)} \]

We chose \( G_{\text{thresh}} = 0.01s^{-1} \) as a threshold value for growth rate, below which the convective sedimentation may not be significant for geophysical applications.
3D Numerical Simulation

- Linear stability analysis can provide some insight on the dynamics of the system, such as the dominant mechanism, growth rate and characteristic finger size.
- Linear stability is only valid for the early stage of the development of instabilities. After the perturbations grow large, interactions between different modes starts to play a role, and the system behaves more nonlinearly.
- 3D numerical simulation can be implemented to investigate the development of instabilities in both linear and nonlinear stage.
- Detailed information, such as sedimentation rate/sediment fluxes, the finger patterns, can be obtained from 3D numerical simulation. These information can be used to derive a semi-empirical or empirical formula for large-scale RANS model.
D = 2 µm  
L = 1 cm

D = 4 µm  
L = 2 cm

D = 20 µm  
L = 10 cm
For very fine particle (2µm) with Λ=0.75, the finger size is on the order of millimeter and double-diffusive mechanism dominates.

\[ V = \frac{\langle w \phi \rangle}{\phi} = \frac{\langle w + V_0 \rangle \phi}{\phi} \]
For relatively larger particle (4µm) with $\Lambda=0.75$, both mechanism are important and the finger size is on the order of centimeter.
For coarse particle (20µm) with $\Lambda=0.6$, settling driven mechanism dominates, the finger size is on the centimeter scale.
Apparent settling velocity

\[ V_0 = 3.6 \times 10^{-4} \text{ cm/s} \]
\[ V = 0.04 \text{ cm/s} \]

\[ V_0 = 1.4 \times 10^{-3} \text{ cm/s} \]
\[ V = 0.4 \text{ cm/s} \]

\[ V_0 = 3.6 \times 10^{-2} \text{ cm/s} \]
\[ V = 1.5 \text{ cm/s} \]

\[ V_0 = 0.32 \text{ cm/s} \]
\[ V = 1.2 \text{ cm/s} \]
Concluding Remarks

- For sediment particle on the order of 2 µm or smaller, double-diffusive mechanism dominates the development of instabilities. The size of sediment finger in this case is on the order of millimeter.
- For sediment particle around 4 µm, the settling-driven mechanism will also play a role. The sediment fingers are very asymmetric with the distance between sediment fingers several times greater than the width of fingers. The finger size is also on the order of centimeter.
- For sediment particle greater than 20 µm, the double diffusive mechanism can be neglected. The size of the sediment finger is on the order of centimeter.
- The enhanced settling due to convective instabilities is more significant for finer particles (clay, fine silt).
- In nature, the convective sedimentation is more important for sediment particle around 20 µm ($V_0=0.036\text{cm/s}$), where the apparent settling velocity is on the order of cm/s.
The roles of rheology in wave-mud interaction

- Previous study (E. Ozdmire et. al 2010) shows for sediment transport of fine particles in wave boundary layer, four different flow regimes exist (sediment as passive scalar, formation of lutocline, partial laminarization and full laminarization) with Re=1000.
- The interplay between turbulent modulation and rheological stresses can alter the transport regimes. By including the effect of rheology, flow laminarization occurs earlier in selected cases.
- A high-accurate spectral-like numerical scheme is implemented to study the effect of rheology to determine the transition of flow regimes and hydrodynamic dissipation.