Rotational Dynamics of Anisotropic Particles in Turbulence

Shima Parsa, Guy Geyer, and Greg Voth
Wesleyan University, USA

Enrico Calzavarini
Université de Lille 1, Lille, France

Federico Toschi
Eindhoven University of Technology, The Netherlands

International Collaboration for Turbulence Research
Why study anisotropic particles in turbulence?

Many applications involve non-spherical particles advected by turbulent flows:

- Dynamics of cellulose fibers in paper industry
- Drag reduction from rods
- Ice crystals in clouds

The rotation rate of a small particles are determined by the velocity gradients which are dominated by the nearly universal small scales.

Rotations represent the Lagrangian history of the velocity gradients along the rod trajectory.

Figure 1.18 Categorization of snow crystals by temperature and excess vapor density.
Lamb and Verlinde (2011)
Experimental Tracking of Rod Trajectories

Stereoscopic imaging allows measurement of rod position and orientation as a function of time. We use 4 cameras with real time image compression. (1024 X 1280 @ 500 Hz)

Turbulent flow between oscillating grids:
Octagonal tank 1x1x1.5 m
Grid mesh size = 8cm

Rods are fluorescently dyed Nylon fibers
d=200 µm and cut to L=1mm
Density, ρ=1.15 g/cm³ with fluid density matched with dissolved CaCl₂.

<table>
<thead>
<tr>
<th>Grid Frequency (Hz)</th>
<th>ν (mm²/s³)</th>
<th>$R_λ$</th>
<th>$ε(=\bar{u}^3/L)$</th>
<th>η = ($ν^3/ε$)¹/⁴</th>
<th>$τ_η = (ν/ε)^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.8</td>
<td>161</td>
<td>354</td>
<td>0.375</td>
<td>71</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>214</td>
<td>2940</td>
<td>0.211</td>
<td>25</td>
</tr>
</tbody>
</table>
An image from one of the four cameras.

Rod concentration is very small: 1 or 2 rods in $(5\text{cm})^3$

The orientation is found in a 2D image. That orientation and the camera viewing direction define a plane in space that contains the rod.

When orientations are measured from at least 3 cameras, we can determine the rod orientation in 3D space.
Time-resolved experimental measurements of motion of rods in 3D lab coordinates

![Graphs showing motion of rods in 3D coordinates](image)
A Rod Trajectory

\[ \lambda = 214 \]

Diameter \( d = 0.2 \text{ mm} = 0.95 \eta \)

Length \( L = 1 \text{ mm} = 4.7 \eta \)

Aspect ratio \( \alpha = L/d = 5 \)

Track duration: \( 284 \text{ ms} = 11 \tau_\eta \)
Direct Numerical Simulation of Rod Trajectories

• DNS simulation of homogeneous turbulence for which the velocity gradient tensor was stored along Lagrangian trajectories.

• Grid of size: \( N^3 = 512^3 \) \( R_\lambda = 180 \)

• \( N = 1.3 \times 10^5 \) particles are followed for \( T = 281 \tau_\eta \)

• Tracer rods (\( L < \eta \)) are advected by the velocity field and their rotational dynamics is integrated using Jeffery’s equation (1922)

\[
\dot{p}_i = \Omega_{ij} p_j + \frac{\alpha^2 - 1}{\alpha^2 + 1} \left( S_{ij} p_j - p_i p_k S_{kl} p_l \right)
\]

\( \vec{p} \) is the orientation of the ellipsoid

\( \alpha \) is the aspect ratio

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

\( \Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \)

strain rate tensor

rotation rate tensor
Tracer Rods properties

- The simplest class of anisotropic particles are axisymmetric ellipsoids. They are parameterized by their aspect ratio:

\[ \alpha = \frac{\text{Length}}{\text{diameter}} \]

\( \alpha > 1 \) prolate ellipsoid, rod shape
\( \alpha < 1 \) oblate ellipsoid, disk shape

- Rods up to 7\( \eta \) rotate as tracer particles

  Shin and Koch  JFM (2005)

- In the experiment we used rods with L<5\( \eta \)

- Particles used in the simulation are tracers (L<<\( \eta \))
Probability Distribution of Rod Rotation Rate

Aspect Ratio, $\alpha=5$
PDF Comparison with enstrophy and dissipation

\[
\begin{align*}
\hat{p}_i \hat{p}_i / \langle \hat{p}_i \hat{p}_i \rangle, \quad \omega^2 / \langle \omega^2 \rangle, \quad \varepsilon / \langle \varepsilon \rangle
\end{align*}
\]
Mean square rotation rate as a function of aspect ratio
Mean Square Rotation Rate for Randomly Oriented Particles

\[ \dot{p}_i = \Omega_{ij}p_j + \frac{\alpha^2 - 1}{\alpha^2 + 1} \left( S_{ij}p_j - p_i p_k S_{kl} p_l \right) \]  

Jeffery's equation

Squaring and taking the mean:

\[ \langle \dot{p}_i \dot{p}_i \rangle = \left( \Omega_{ij} \Omega_{im} p_j p_m \right) + \left( \frac{\alpha^2 - 1}{\alpha^2 + 1} \right)^2 \left( \langle S_{ij} S_{im} p_j p_m \rangle + \langle p_i p_k S_{kl} p_l p_i p_q S_{qn} p_n \rangle \right) \]

+ 6 other cross terms

If rods are randomly oriented, then

\[ \langle S_{ij} S_{im} p_j p_m \rangle = \langle S_{ij} S_{im} \rangle \langle p_j p_m \rangle \] etc.

\[ \langle \dot{p}^2 \rangle = \frac{1}{3} \langle \Omega_{ij} \Omega_{ij} \rangle + \frac{1}{5} \left( \frac{\alpha^2 - 1}{\alpha^2 + 1} \right)^2 \langle S_{ij} S_{ij} \rangle \]

\[ \frac{\dot{p}^2}{\nu} = \frac{1}{6} + \frac{1}{10} \left( \frac{\alpha^2 - 1}{\alpha^2 + 1} \right)^2 \]

using the relation for isotropic turbulence

\[ \langle S_{ij} S_{ij} \rangle = \langle \Omega_{ij} \Omega_{ij} \rangle = \frac{\varepsilon}{2 \nu} \]
Mean square rotation rate as a function of aspect ratio

\begin{align*}
\langle \dot{\epsilon} \rangle = \nu \frac{10^{-2}}{10^{-1}}
\end{align*}
Mean square rotation rate for simple velocity gradient models

Simple models do not even qualitatively capture the dependence of rotation rate on aspect ratio. Statistics and time correlations of velocity gradients are important.
Probability Distribution of Rod Rotation Rate

Probability distribution

\[ \frac{\dot{p}_i \dot{p}_i}{\langle \dot{p}_i \dot{p}_i \rangle} \]

Aspect Ratio, \( \alpha = 5 \)
Dependence of the tail of the rotation rate PDF on aspect ratio

Fourth moment shows much weaker dependence on aspect ratio than the second moment, indicating that alignment effects are the result of typical flow structures and not rare events.

Ongoing Work:

Measuring rotations of spheres and disks: 3D printed jacks rotate like spheres and crosses rotate like disks. Jacks allow measurement of Lagrangian Vorticity and helicity.

Simulations can obtain rotations of inertial anisotropic particles.

Measuring rotations of rods longer than the Kolmogorov length.

Resolving the velocity gradient tensor along particle trajectories.
Conclusions

- Careful stereoscopic imaging can produce accurate time resolved measurements of the orientations of anisotropic particles in intense turbulence.
- Very good agreement between experiments and DNS.
- DNS of trajectories of rods resolves the details of dependence of rotation rate on the aspect ratio.
- Rotation rate of rods depends strongly on alignment with the velocity gradient tensor along the rod trajectory.

Contact: gvoth@wesleyan.edu
randomly oriented

After advection by the flow partially aligns rods with the strain rate

Dependence of Rotation Rate on Rod Length

Shin and Koch  JFM (2005)
Measuring rotation rate from experimental measurements of rod orientation

- We determine the variance of the rotation rate by extrapolating to zero fit-time from the measurements at different fit-length.

- Choosing an appropriate fit-length on the data is crucial for measuring the rotation rate accurately.