Particles in turbulence

Federico Toschi

NCAR, Boulder CO, 14 August 2012
• Introduction

• Motivation to study particles in turbulence

• Short review of results on point particles

• A “point particle” model for finite size particles

• Fully deformable droplets...

• Conclusions
Dictionary...

- **Tracer**: a tiny particle moving following the streamlines of the flow (e.g. a fluid molecule)
- **Heavy** particle:
  - density $\gg$ fluid density
  - size $\ll$ dissipative scale
- **Light** particle:
  - density $\ll$ fluid density
  - size $\ll$ dissipative scale
- **Large** particle:
  - $D \leq 10 \eta$
- **Deformable** “particle”
  - deforming droplets with surface tension
Where is Lagrangian turbulence?

Plankton

Pollution

Dust storms

Combustion
Leonardo da Vinci; circa 1500 (translated by Ugo Piomelli): “Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion.”

\[
\frac{dx}{dt}(t|x_0, t_0) \equiv u_L(t|x_0, t_0)
\]

\[
u_L(t|x_0, t_0) \equiv u_E(x(t|x_0, t_0), t)
\]
Eulerian turbulence

\[ \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f} \]
\[ \partial \cdot \mathbf{v} = 0 \]

Richardson energy cascade

\[ \text{Re} = \frac{L_0 \nu}{\nu} \]

\( \eta \ll r \ll L_0 \)

Inertial range

\( \eta = (\nu^3/\varepsilon)^{1/4} \)
\( \tau_\eta = (\nu/\varepsilon)^{1/2} \)
The “standard model”

\[
S_p(r) = \langle [\mathbf{v}(\mathbf{x} + r) - \mathbf{v}(\mathbf{x})]^p \rangle \\
S_p(r) = \langle (\delta_r \mathbf{v})^p \rangle \sim \langle \mathbf{v}_0^p \rangle \int_I dh \left( \frac{r}{L_0} \right)^{hp + 3 - D(h)}
\]

\[
S_p(r) \sim \left( \frac{r}{L_0} \right)^{\zeta_p}
\]

\[
\zeta_p = \inf_h (hp + 3 - D(h))
\]

Multi-fractal model
Parisi-Frisch 1995

\[
\eta \ll r \ll L_0
\]
Tracers acceleration

\[ a \equiv \frac{\delta v(\tau_\eta)}{\tau_\eta} \]

FIG. 1: Trajectory and time series. Left panel: 3D trajectory of a trapping event in vortex filament. Acceleration and velocity fluctuations here reach about 30 and 2 r.m.s. values, respectively (right panels).
**Acceleration multi-fractal view**

\[
a = \frac{\delta_{\tau h} v}{\tau h}
\]

with probability

\[
(\tau_{h, v_0} / T_L(v_0))^{(3 - D(h)) / (1 - h)}
\]

\[
\mathcal{P}(v_0) = \exp \left( -\frac{v_0^2}{2\sigma^2} \right) / \sqrt{2\pi\sigma^2}
\]

The small scale fluctuates !!!

The large scale fluctuates !!!
Acceleration multi-fractal view

From standard multifractal arguments:

\[
P(a^2) \sim \int_{h \in I} dh \, a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \exp \left( -\frac{a^{2(1-h)}}{3} \nu^{\frac{2(1-2h)}{3}} \frac{L_0^{2h}}{2\sigma_v^2} \right)
\]

Supposing without intermittency (K41 case)

\[
P^{K41}(\tilde{a}) \sim \tilde{a}^{-5/9} R_{\chi}^{-1/2} \exp \left( -\tilde{a}^{8/9}/2 \right)
\]

...with h=1/3

Also able to make other predictions,
i.e. acceleration variance conditioned to velocity value:

\[
\left\langle a^2 | v_0 \right\rangle \sim \int_{h \in I} dh \, \nu^{\frac{1+4h-D(h)}{1+h}} v_0^{\frac{3+D(h)}{1+h}} L_0^{\frac{D(h)-6h-3}{1+h}}
\]

L. Biferale, G. Boffetta, A. Celani, B. Devenish, A. Lanotte, and F. Toschi.
Multifractal statistics of lagrangian velocity and acceleration in turbulence.
Acceleration p.d.f. result for tracers

\[ P(\tilde{a}) \]

\[ \tilde{a} = a / \sigma_a \]

K41 prediction

Multifractal prediction
Lagrangian velocity statistics

We do not have much fantasy! So we “copy” from Eulerian turbulence: structure functions

\[ S_p(r) \equiv \langle (v(x + r) - v(x))^p \rangle \sim r^\zeta_E(p) \]
\[ S_p(\tau) \equiv \langle (v(t + \tau) - v(t))^p \rangle \sim \tau^\zeta_L(p) \]

\[ \tau_\eta \ll \tau \ll T_L \]

\[ S_2(\tau) = c_0 \varepsilon \cdot \tau \]

Does it exist and how to estimate \( \zeta_L(p) \) ?

In Eulerian turbulence we have \( \zeta_E(p) = \inf_{h}(hp + 3 - D(h)) \)
The local exponents $\zeta_p(\tau)$ act as magnifying glass, probing locally the value of the scaling exponents. By comparing with $S_2$ we also take advantage of the Extended Self Similarity (ESS).

Power law scaling \[ \text{plateaux} \] for local scaling exponents

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<tr>
<th>No.</th>
<th>$R_\lambda$</th>
<th>$v'_{rms}$ (m/s)</th>
<th>$\varepsilon$ ($m^2/s^3$)</th>
<th>$\eta$ (µm)</th>
<th>$\tau_\eta$ (ms)</th>
<th>$T_L$ (s)</th>
<th>$N_f$ ($f/\tau_\eta$)</th>
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$S_2(\tau) \sim \varepsilon \cdot \tau$
Local scaling exponents

\[ \tilde{\zeta}_p(\tau) = \frac{d \log (S_p(\tau))}{d \log (S_2(\tau))} \]

DNS and experiment show a very good agreement
MF: Lagrangian velocity statistics

We assume that $\tau$ and $\mathcal{R}$ are linked by the typical eddy turn over time at the given spatial scale

$$\tau_r \sim r / \delta_r u$$

Bridge between eulerian and lagrangian description:

$$\delta_r u \sim \delta_r u \quad \text{and} \quad \tau \sim \frac{L_h}{v_0} r^{1-h}$$
MF: Lagrangian structure functions

Multifractal prediction for the Lagrangian structure functions

\[ S_p(\tau) \sim \langle v_0^p \rangle \int_{h \in I} dh \left( \frac{\tau}{T_L} \right)^{\frac{hp + 3 - D(h)}{1 - h}} \]

where

\[ \zeta_L(p) = \inf_h \left( \frac{hp + 3 - D(h)}{1 - h} \right) \]

This allow us to actually predict the following value:

\[ \frac{\zeta_L(4)}{\zeta_L(2)} = 1.71 \quad \frac{\zeta_L(6)}{\zeta_L(2)} = 2.16 \quad \frac{\zeta_L(8)}{\zeta_L(2)} = 2.72 \]
Velocity structure functions (tracers)

Eulerian (space) -> Lagrangian (time)

\[ \delta_r v = v_0 \cdot \left( \frac{r}{L_0} \right)^h \]

\[ \delta_\tau v \sim \left( \frac{\tau}{T_L} \right)^{\frac{h}{1-h}} \]

\[ \delta_\tau v = v_0 \cdot \left( \frac{\tau}{T_L} \right) \cdot \left[ \left( \frac{\tau}{T_L} \right)^\beta + \left( \frac{\tau \eta}{T_L} \right)^\beta \right]^{\frac{2h-1}{\beta(1-h)}} \]

\[ \delta_\tau v \sim \left( \frac{\tau}{T_L} \right)^{\frac{h}{1-h}} \]

\[ \delta_r v \propto \tau \]

Start from Eulerian \( D(h) \)

\[ \beta = 4 \]
Lagrangian turbulence IS universal


\[ \zeta(4, \tau) \]

\[ \tau/\tau_\eta \]

EXP1 $Re_\lambda = 124$
EXP2 $Re_\lambda = 690$
EXP3 $Re_\lambda = 740$
DNS1 $Re_\lambda = 140$
DNS2 $Re_\lambda = 320$
DNS3 $Re_\lambda = 400$
DNS4 $Re_\lambda = 600$
DNS5 $Re_\lambda = 650$
How to model (point) particles in turbulence?
Faxen pointwise particle model

\[
\frac{d\mathbf{v}}{dt} = \beta \left[ \frac{D\mathbf{u}}{Dt} \right]_V + \frac{3\nu\beta}{r_p^2} ([\mathbf{u}]_S - \mathbf{v}) + \frac{3\beta}{r_p} \int_{t-t_h}^{t} \left( \frac{\nu}{\pi(t-\tau)} \right)^{\frac{1}{2}} \frac{d}{d\tau} ([\mathbf{u}]_S - \mathbf{v}) \, d\tau \\
+ c_{Re_p} \frac{3\nu\beta}{r_p^2} ([\mathbf{u}]_S - \mathbf{v}) + \left( 1 - \frac{3\rho_f}{\rho_f + 2\rho_p} \right) \mathbf{g}
\]

\[
\left[ \frac{D\mathbf{u}}{Dt} \right]_V = (4/3 \pi r_p^3)^{-1} \int_V \frac{D\mathbf{u}}{Dt} (\mathbf{x}, t) \, d^3 \mathbf{x}
\]

\[
[\mathbf{u}]_S = (4\pi r_p^2)^{-1} \int_S \mathbf{u}(\mathbf{x}, t) \, d^2 \mathbf{x}
\]
Numerical study of particles in turbulence

Bubbles $\beta=3$, $St=0.6$ and $Re_\lambda=78$
Moore law for DNS of HI turbulence

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<th>Year</th>
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<th>Rλ</th>
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<td>1972</td>
<td>32</td>
<td>35</td>
<td>Orszag, Patterson, PRL 28 76</td>
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<td>Vincent, Meneguzzi, JFM 25 1</td>
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<td>Sanada, PRA 44 6480</td>
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<td>She et al., PRL 70 3251</td>
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<td>Gotoh, Fukuyama, PRL 86 3775</td>
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<td>Kaneda et al. Phys.Fluids 15 L21</td>
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<td>2006</td>
<td>4096</td>
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<td>Kaneda et al. J. Turb. 7 20</td>
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Floating Point Operation for eddy turnover time:
FLOP \( \propto N^3 \log(N) \times N \) (at high N flop=660 N^3 log(N) for time step)
Memory: ram \( \propto N^3 \) (at high N, RAM=182 N^3 byte)

\[ \text{best fit: } \text{Re}_\lambda = 2.26 \, N^{0.75} \]

\[ K_{41}: \text{Re}_\lambda \approx N^{2/3} \]
CINECA keyproject 1024³ DNS+tracers

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<th>N</th>
<th>Reₜ</th>
<th>η</th>
<th>L</th>
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Pseudo spectral code - dealiased 2/3 rule - normal viscosity - 2 millions of passive tracers- code fully parallelized with MPI +FFTW - Platform IBM SP4 (sust. Performance 150Mflops/proc) - 50000 cpu hours - duration of the run: 40 days

Lagrangian database
(x(t),v(t),a(t)=-∇p+νΔu)
with high temporal resolution
Pseudo spectral code - dealiased 2/3 rule - normal viscosity - 2 billions of passive tracers & heavy particles - code fully parallelized with MPI+FFTW - Platform SGI Altix 4700 - 400000 cpu hours - duration of the run: 40 days over 3 months.

Lagrangian database \((x(t),v(t),u(t),\partial iu_j(t))\) at high resolution
# CASPUR $512^3$ DNS tracers & heavy & light

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<th>Re$_\lambda$</th>
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<th>$L$</th>
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Pseudo spectral code - dealiased 2/3 rule - normal viscosity - 100 millions of passive tracers & heavy/light particles - code fully parallelized with MPI+FFTW - Platform IBM SP5 1.9 GHz - 30000 cpu hours - duration of the run: 30 days.

64 different particles classes ($\beta, St$)

Lagrangian database $(x(t), v(t), u(t), \partial_{ij} u(t))$ at high resolution

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### Heavy particles - parameters

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<td>Inverse cascade turbulence</td>
<td>Guido Boffetta</td>
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<td>Lagrangian tracers in 3D homogeneous and isotropic turbulent velocity field</td>
<td>Federico Toschi</td>
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<td>Database of Particles Dispersed in a Stirred-Tank Reactor</td>
<td>Valentina Lavezzo</td>
<td>57.97 MB</td>
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</tbody>
</table>
All effects of inertia, in a slide...

Preferential concentration

Filtering of turbulent fluctuations

Bubbles

Tracers

Heavy

\[ P(a/a_{rms}) \]

\[ a_{rms}, \langle l^2 \rangle^{1/4} \]

Acceleration at varying Stokes no model

- Fluid acceleration at particle’s position
- Filtered acceleration
- Heavy particle’s acceleration

\[ u^f(t) = \frac{1}{\tau_s} \int_{-\infty}^{t} e^{-(t-s)/\tau_s} u(x(s), s) \, ds \]

\[ Re_\lambda = 185 \]
More general picture of forces

\[ a_{\text{rms}} \]

\[ \text{St} = \frac{\tau}{\tau_\eta} \quad \beta = \frac{3\rho_f}{\rho_f + 2\rho_p} \]

\[ \frac{d\mathbf{v}(t)}{dt} = \beta \frac{D\mathbf{u}(x, t)}{Dt} - \frac{1}{\tau} [\mathbf{v}(t) - \mathbf{u}(x(t), t)] \]

Toschi and Bodenschatz. Lagrangian properties of particles in Turbulence.
Preferential concentration

\[ \beta = 0, \text{ heavy} \]
\[ \beta = 1, \text{ tracer} \]
\[ \beta = 3, \text{ bubble} \]

Poisson distribution

bubble

tracer

heavy

\[ \text{St} = 0.6 \]

\[ 10^5 \text{ particles} \]

\[ 33 \]
Kaplan-Yorke dimension: $D_{KY}$

Particle equations of motion defines a dissipative dynamical system
Attractor’s dimension in the $(x,v)$ space: Kaplan-Yorke dimension $D_{KY}$

$$D_{KY} \equiv J - \frac{\lambda_1 + \cdots + \lambda_J}{\lambda_{J+1}}$$

$\lambda_1 + \cdots + \lambda_J \geq 0$

$\lambda_1 + \cdots + \lambda_{J+1} < 0$

6 Lyapunov exponents computed by tracking

$$\mathbf{R}(t) \equiv (\delta x(t), \delta v(t))$$

$$\frac{d\mathbf{R}}{dt} = \mathbf{M}_t \mathbf{R}$$

$$\lambda_i = \lim_{T \to \infty} \gamma_i(T)$$

Standard orthonormalization
Gram-Schmidt procedure adopted

Kaplan-Yorke dimension

Balance between contraction and expansion

\[ D_{KY} \equiv J - \frac{\lambda_1 + \cdots + \lambda_J}{\lambda_{J+1}} \]

**Heavy** min. at
St ≈ 0.5, \( D_{KY} \approx 2.6 \)

**Light** min at
St ≈ 1, \( D_{KY} \approx 1.4 \)

Close to fractal dimension of vortex filaments in turbulence? \( D_{\omega} \approx 1.1 \)
(Moisy & Jimenez JFM 04)

\[ \lambda_1 + \cdots + \lambda_J \geq 0 \]
\[ \lambda_1 + \cdots + \lambda_{J+1} < 0 \]
**Correlation dimension $D_2$**

$P_2(r)$ Probability to find a couple of particle whose distance is below $r$.

At $r \ll \eta$ $P_2(r) = A r$

fractal dimension hierarchy: $D_2 \leq D_1 = D_{KY}$

Similar features as $D_{KY}$
Finite size particles

\[
St = \frac{\tau_p}{\tau_\eta} \quad \beta = \frac{3 \rho_f}{\rho_f + 2 \rho_p}
\]

\[
St = \frac{\tau_p}{\tau_\eta} = \frac{r^2}{\tau_\eta} \left( \frac{\rho_f + 2 \rho_p}{9 \nu \rho_f} \right) = \frac{1}{\tau_\eta} \frac{r^2}{3 \nu \beta}
\]
Motivation: Marine snow

Particle aggregation processes and how they affect particles in the marine environment. Biological aggregation (e.g., fecal pellet production) and physical aggregation by (a) shear and (b) differential sedimentation form large, heterogeneous, rapidly settling particles in the surface waters. In deeper waters, fragmentation and repackaging of this material by zooplankton are the dominant processes that affect aggregate sizes and properties. Microbes decompose material throughout the water column.

Bacteria: small scale view
Bacteria: large scale view

http://earthobservatory.nasa.gov/IOTD/view.php?id=41385
Plankton: small scale view
Plankton: large scale view

"Van Gogh" Algae
Image courtesy EROS/USGS/NASA

In the style of Van Gogh's "Starry Night," massive congregations of greenish phytoplankton swirl in dark water around Sweden's Gotland (see map) island in a satellite picture released this week by the U.S. Geological Survey (USGS).

The image of the Baltic Sea island is 1 of 40 in the new Earth as Art 3 collection, the latest compilation of Landsat pictures chosen for their artistic quality.

"The collected images are authentic and original in the truest sense," Matt Larsen, the USGS's associate director for Climate and Land Use Change, said in a statement. "These magnificently engaging portraits of Earth encourage us all to learn more about our complex world.”

Population explosions, or blooms, of phytoplankton, like the one shown here, occur when deep currents bring nutrients up to sunlit surface waters, fueling the growth and reproduction of these tiny plants, according to the USGS.

(Related: "The Best Pictures of Earth: Reader Picks of NASA Shots.")

Published November 19, 2010
Recent experiments on acceleration statistics with $D \geq \eta$ particles:

- N. Qureshi et al. EPJ B (2008)
- R. Volk et al. JFM (2011)

- Experimentally most studied case: neutrally buoyant particles.
Instrumented particle


Instrumented tracer for Lagrangian measurements in Rayleigh-Bénard convection
Tracking the dynamics of translation and absolute orientation of a sphere in a turbulent flow
Robert Zimmermann, Yoann Gasteuil, Mickael Bourgoin, Romain Volk, Alain Pumir, and Jean-Francois Pinton
Finite size particles

These clear, colorless spheres made of sodium polyacrylate - the superabsorbent polymer found in diapers - grow from 4 to 20 mm in diameter. Hydrated spheres have the refractive index of water, so they are invisible in water. Students can monitor growth rate or calculate before and after volumes as spheres hydrate over 12 hr.

Each pack contains 50 g (about 1,100 spheres). Re-use objects multiple times. Allow 12 hr to air dry.
Abstract: Flow around finite-size neutrally buoyant Lagrangian particles in fully developed turbulence DFD 2010

Mathieu Gibert
(Max Planck Institute for Dynamics and Self-Organization)

Simon Klein
(Max Planck Institute for Dynamics and Self-Organization)

Antoine B'erut
(Max Planck Institute for Dynamics and Self-Organization)

Eberhard Bodenschatz
(Max Planck Institute for Dynamics and Self-Organization)

By using an innovating technique based on Lagrangian Particle Tracking (LPT), we have been able to follow the motion of finite-size neutrally buoyant particles together with the trajectories of tracer particles in the surrounding fluid. The particles we study have diameters of about 200 times the dissipative scale of the flow, and their density is almost that of the fluid. The experiments are conducted in a von Karman swirling water flow at Taylor microscale Reynolds numbers up to 500. By measuring the full motion of the big particles (translation and rotation), we are able to ``sit'' in their frame of reference and measure the flow properties around them. We will report experimental results on the flow properties and its correlations with the big particle trajectories in this Lagrangian frame.
Figure 1. (Colour online) Experimental set-up. (a) Geometry of the turbulence generator. (b) Schematics of the von Karman flow in water. (c) Principle of the LDV using wide beams (eLDV) – top view of the experiment. PM denotes location of the photomultiplier which detects scattering light modulation as a particle crosses the interference pattern created at the intersection of the laser beams. The eLDV measures, for one particle at a time, the evolution of its velocity component $u_x(t)$ along the particle trajectory.

Phenomenology (neutrally buoyant particle)

For $D > \eta$, acceleration variance decreases when $D$ is increased

$$\left\langle a^2 \right\rangle \sim \varepsilon^{4/3} D^{-2/3}$$

Voth et al. (2002)

Qureshi et al. (2007)
Phenomenology (neutrally buoyant particle)

Early experiments were showing acceleration PDF very weakly dependent on $D$ and non-Gaussian, recently things changed.
From the Point Particle model (PP)...

Point-like heavy/neutral/light inertial particles: Stokes drag, fluid acceleration & added mass

\[
\frac{d\mathbf{v}}{dt} = \frac{3 \rho_f}{\rho_f + 2 \rho_p} \left( \frac{D\mathbf{u}}{Dt} + \frac{3\nu}{a^2} (\mathbf{u} - \mathbf{v}) \right)
\]

\[
\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}
\]

\[
\tau_p = \frac{1}{3\beta} \frac{a^2}{\nu}
\]

\[
St = \frac{\tau_p}{\tau_\eta} = \frac{1}{3\beta} \frac{a^2}{\eta^2}
\]

0 ≤ β ≤ 3

dilute suspension (no collisions), no particle feedback on the flow
A minimal model for finite-sized particles: the Faxén Corrected model (FC)

\[
\frac{dv}{dt} = \beta \left[ \frac{Du}{Dt} \right]_v + \frac{3 \nu \beta}{r_p^2} ([u]_S - v)
\]

Accounts for the nonuniformity of the flow at the particle scale

\[ [u]_S = \left(4\pi r_p^2 \right)^{-1} \int_S u(x, t) \, d^2 x \]

\[ \frac{Du}{Dt}_v = \left(4/3 \pi r_p^3 \right)^{-1} \int_V \frac{Du}{Dt}(x, t) \, d^3 x \]

\[ \rho_f, \nu \]

\[ \rho_p, a \]

\[ 2r \]

Acceleration statistics of finite-sized particles in turbulent flow: the role of Faxen forces
Efficient numerical implementation

**Approximation:**

Sphere Average $\rightarrow$ Gaussian Average

\[
\langle u_i \rangle_{G,V(a)}(x) = \mathcal{DFT}^{-1}_{(N^3)} \left[ \tilde{G}_\sigma(k) \tilde{u}_i(k) \right]
\]

\[
\tilde{G}_\sigma(k) = e^{-\frac{1}{2} \sigma^2 k^2}
\]

\[
\langle u \rangle_{S_{a}} = \frac{1}{3a^2} \frac{d}{da} \left( a^3 \langle u \rangle_{V(a)} \right)
\]

\[
\langle u_i \rangle_{G,S(a)}(x) = \mathcal{DFT}^{-1}_{(N^3)} \left[ \tilde{S}_a(k) \tilde{u}_i(k) \right]
\]

with $\sigma = a/\sqrt{5}$. $\rightarrow$ first order Faxén corrections
PP model (neutrally buoyant particles)

- Acceleration variance is $D$ independent
- Acceleration PDF is $D$ independent
- Correlation time of Acceleration does not grow with $D$, but reduces.

See:

\[
\frac{a_{\text{rms}}}{\langle D_t u \rangle_{\text{rms}}} = 1 \quad \text{same acc. as the fluid}
\]

$St = 4$
$D/\eta \approx 7$
Acceleration Variance from:
Point-Particle and Faxén corrected models

\[ \frac{\langle a^2 \rangle}{\langle a_f^2 \rangle} \]

DNS (128^3) \( Re_\lambda = 75 \)

- **light**
- **neutral**
- **heavy**

- \( \rho_p/\rho_f = 10 \), filter
- no filter
- \( \rho_p/\rho_f = 1 \), filter
- no filter
- \( \rho_p/\rho_f = 0.1 \), filter
- no filter

/Federico Toschi
Acceleration Flatness

\[ F(a_f) \rightarrow F(a_f) \]

DNS (128^3) \( \text{Re}_\lambda = 75 \)

\[ D/\eta \]

FC \( \rho_p/\rho_f = 10 \)
PP

FC \( \rho_p/\rho_f = 1 \)
PP

FC \( \rho_p/\rho_f = 0.1 \)
PP

light

neutral

heavy
PDF of acceleration: exp vs. FC simulations

DNS ($512^3$) $Re_\lambda = 180$
EXP (Qureshi et al. 2007) $Re_\lambda = 160$

EXP (Volk et al. 2011)
Acceleration Variance: comparison

\[ a_0 = \langle a^2 \rangle \varepsilon^{-3/2} v^{1/2} \]

\[ \text{(Voth et al. 2002)} \]
\[ \text{(Qureshi et al. 2007)} \]

EXP* Re_\lambda = 970
EXP Re_\lambda = 160
DNS Re_\lambda = 180
DNS Re_\lambda = 75
\[ D^{-2/3} \]
Acceleration Variance: comparison

(Voth et al. 2002) EXP* Reₜ = 970
(Qureshi et al. 2007) EXP Reₜ = 160
DNS Reₜ = 180
DNS Reₜ = 75

\[ a_0 = \langle a^2 \rangle \varepsilon^{-3/2} \nu^{1/2} \]

\[ \frac{\langle a^2 \rangle}{\langle q_f^2 \rangle} \]

\[ \frac{D}{\eta} \]

\[ \frac{1}{100} \]
More refined FC model for finite size particles

Impact of trailing wake drag on the statistical properties of finite-sized particles in turbulence

More refined particle model

\[
\frac{dv}{dt} = \beta \left[ \frac{Du}{Dt} \right]_V + \frac{3\nu \beta}{r_p^2} ([u]_S - v) \quad \text{(Faxen corrected model with Stokes drag)}
\]

\[
+ \frac{3\beta}{r_p} \int_{t-t_h}^t \left( \frac{\nu}{\pi(t-\tau)} \right)^{\frac{1}{2}} \frac{d}{d\tau} ([u]_S - v) \, d\tau \quad \text{(History force (unsteady Stokes drag))}
\]

\[
+ c_{Re_p} \frac{3\nu \beta}{r_p^2} ([u]_S - v) \quad \text{(Non-Stokesian drag force (model for the wake drag))}
\]

\[
c_{Re_p} = 0.15 \cdot Re_p^{0.687} \quad \text{Schiller-Naumann (1933)}
\]

\[
Re_p \equiv \|[u]_S - v\| d_p / \nu \quad \text{Re}_p < 1000
\]
Particle Reynolds number

Focus on the case $Re_\lambda \sim 31$ and neutrally buoyant particles to compare with fully resoled DNS by Homann & Bec 651 81-91 J. Fluid Mech. (2010)
Velocity variance

\[ \frac{\langle u_i^2 \rangle}{\langle u_i \rangle^2} - \frac{\langle u_i^2 \rangle}{\langle u_i \rangle^2} \]

- Faxen model with Stokes drag
- + Non-Stokesian drag
- + History
- + Non-Stokesian drag + History

\[ d_f^2/(12 \nu <u^2>) \]

1 - \[ \langle |u|^2 \rangle \langle u^2 \rangle \]

Homann and Bec (2010)

\[ v = \langle |u| \rangle_s \]

Particle

Fluid

\[ d_p/\eta \]
Acceleration variance

\[ a = \left[ \frac{Du}{Dt} \right] v \quad \text{(sphere)} \]

\[ a = \left[ \frac{Du}{Dt} \right] v \quad \text{(Gauss)} \]

(with improved volume averages)
Large Eddy Simulation

- Filtered velocity field
- Filtered energy spectra

Resolved scales
Unresolved scales

/ TN & WI

Federico Toschi

Technische Universität Eindhoven University of Technology
Effects of droplet deformability
Droplets deformability: key issues

- Physics: finite size particles plus surface tension
- Transfer of energy from fluid to elastic modes (and vice versa)
- How is turbulence affected by the presence of droplets?
- How do properties of (deformable) droplets differ from rigid droplets?
FIG. 1 (color online). Reconstructed holographic images showing turbulent stretching of a crude oil droplet for DOR of 1:20, with inset showing the capillary breakup of a section 2 ms later.

Experimental investigations

Prakash, Tagawa, Martinez-Mercado, Sun, Lohse
Experiment in the Twente water tunnel.

Bubbles deform in presence of flows.
Some recent numerical work

Some recent numerical work

Figure 2. Demixing simulation. Cross sections through a fully periodic $128^3$ domain. Time proceeds from left to right, the time interval between subsequent images being 500 time steps.

Figure 6. DSD (by number of drops) at (from left to right) $t = 13 \tau_\kappa$, $26 \tau_\kappa$, $39 \tau_\kappa$, $52 \tau_\kappa$, and $65 \tau_\kappa$.

Dimensionless numbers

- Turbulence
- Inertial force
- Surface tension force
- Weber number

\[ Re = \frac{u'L}{\nu} \]
\[ Re_d = \frac{u_d d}{\nu} \]
\[ Ca = \frac{\mu u_d}{\sigma} \]
\[ We = \frac{\rho u_d^2 d}{\sigma} \]

*J.O. Hinze, A.I.Ch.E, (1955)*
Elongation in a stationary flow

\[ D = \frac{19\lambda + 16}{16\lambda + 16} \cdot Ca \]

\[ Ca = \frac{\mu u_d}{\sigma} \]

\[ D = \frac{R_1 - R_2}{R_1 + R_2} \]

\[ \lambda = 1 \]

H. Stone, “Dynamics of drop deformation and breakup in viscous fluids”
\[ W_e = \frac{\rho u_d^2 d}{\sigma} \]

K41

\[ u_d^2 \sim d^{2/3} \varepsilon^{2/3} \]

\[ d > d_{\text{max}}: \text{Droplet breaks} \]
\[ d < d_{\text{max}}: \text{Droplet does not break} \]

\[ d_{\text{max}} = 0.75 \left( \frac{\rho}{\sigma} \right)^{-3/5} \varepsilon^{-2/5} \]

Fig. 6. Maximum drop size as a function of the energy input according to experimental data by Clay.
Numerical approach

Lattice Boltzmann Method (LBM)

D3Q19 BGK LB model

\[ f_\alpha(x + e_\alpha, t + 1) = f_\alpha(x, t) - \frac{f_\alpha(x, t) - f_\alpha^{(eq)}(x, t)}{\tau} \]

with multicomponent Shan-Chen

Technique inspired to the continuum Boltzmann equation

\[ f \equiv f(x, v, t) \]

\[ \partial_t f + (v \cdot \nabla) f = \Omega - (F \cdot \nabla) f \]
\[ f^{\beta}_{\alpha}(x + c_{\alpha}, t + 1) = f^{\beta}_{\alpha}(x, t) - \frac{1}{\tau_{\beta}} \left[ f^{\beta}_{\alpha}(x, t) - f^{eq\beta}_{\alpha}(x, t) \right] \]

\[ u^{\beta}(x, t) = u^{\beta}(x, t) + \frac{\tau F(x, t)}{\rho^{\beta}} \]

\[ F^{\alpha\beta} = -G \rho^{\alpha}(x) \cdot \sum_{\gamma} \rho^{\beta}(x + e_{\gamma}) \]

\[ \rho = \sum_{\beta} \rho^{\beta} \quad \rho u = \sum_{\beta} \rho^{\beta} u^{\beta} \]

**Convincing LBM to go turbulent**

Forcing: Large scale forcing in first two Fourier modes

\[
\begin{align*}
  f_x &= \sum_{k \leq \sqrt{2}} f_0 [\sin(k_y y + \phi_k^2) + \sin(k_z z + \phi_k^3)] \\
  f_y &= \sum_{k \leq \sqrt{2}} f_0 [\sin(k_x x + \phi_k^1) + \sin(k_z z + \phi_k^3)] \\
  f_z &= \sum_{k \leq \sqrt{2}} f_0 [\sin(k_x x + \phi_k^1) + \sin(k_y y + \phi_k^2)]
\end{align*}
\]

For a simulation with the given parameters:

- \( N = 512^3 \)
- \( \nu = 5 \times 10^{-3} \)
- \( \lambda \approx 13.89lu \)
- \( \eta \approx 6lu \)
- \( \sigma \approx 0.028 \)
- \( Re_\lambda \approx 29.13 \)

Random phases generated from Ornstein-Uhlenbeck process
LBM: Energy and enstrophy

\[ E = \frac{1}{2} \int \rho u^2 \]
\[ \Omega = \int \omega^2 \]
LBM: Energy and acceleration

Acceleration of a fluid parcel

\[ a = \frac{D u}{D t}; \quad a \approx -\nabla p \approx -c_s^2 \nabla \rho \]

\[ Re_\lambda \approx 300 \]

SC: acceleration for single component flow
Simulations

- JUGENE (FZJ-JSC IBM Blue Gene/P)
- 23.5RM (about 15Mhours)
- 32-64 kprocs
- I/O HDF5
- Fully parallel code
Droplet breakup in turbulence
Towards a stationary state...
Droplet radius vs. time

\[ \phi = 0.5\% \quad \phi = 5\% \quad \phi = 10\% \]
### Simulation parameters

<table>
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<th>Re</th>
<th>N</th>
<th>density ratio (liquid / saturated vapor)</th>
<th>viscosity</th>
<th>G</th>
<th>dissipative scale (LU)</th>
<th>D Hinze</th>
<th>D (LBE)</th>
<th>We</th>
<th>Volume fraction %</th>
<th>Eddy turnover</th>
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<td>1.757/0.08</td>
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<td>0.08</td>
<td>6</td>
<td>39.5</td>
<td>36+/-1</td>
<td>0.033</td>
<td>0.3%</td>
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<td>128</td>
<td>0.5/0.5</td>
<td>0.005</td>
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<td>3</td>
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<td>50, 50, 560</td>
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pdf of radius and volumes
Droplet PDF: Re dependence

<table>
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<th>$Re_\lambda$</th>
<th>$d_{32}$ (LBM)</th>
<th>$\varepsilon$</th>
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<tr>
<td>40</td>
<td>14.5</td>
<td>$3.5 \times 10^{-8}$</td>
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<tr>
<td>80</td>
<td>24</td>
<td>$2.85 \times 10^{-9}$</td>
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</table>
Verification of Hinze criterion

\[ \rho \sigma \langle D \rangle = 1.6 \times 10^{-3}, \phi = 0.3\%, N512A \]
\[ \rho \sigma \langle D \rangle = 1.7 \times 10^{-3}, \phi = 0.3\%, N512B \]
\[ \rho \sigma \langle D \rangle = 1.6 \times 10^{-3}, \phi = 0.07\%, N128A \]
\[ \rho \sigma \langle D \rangle = 1.6 \times 10^{-3}, \phi = 0.5\%, N128B \]
\[ \rho \sigma \langle D \rangle = 1.6 \times 10^{-3}, \phi = 5\%, N128C \]
\[ \rho \sigma \langle D \rangle = 1.6 \times 10^{-3}, \phi = 10\%, N128D \]

Fig. 6. Maximum drop size as a function of the energy input according to experimental data by Clay.
Trajectories look smooth but acceleration is very noisy
PDF of acceleration
Droplet deformation

- **Surface S**
  - $S(t)$
  - $S = 4\pi R^2$

- **Volume V**
  - $S_0$
  - $V = \frac{4}{3}\pi R^3$

Deformation parameter is the dimensionless surface i.e. normalized wrt the surface of the equivalent sphere with same volume V.
Deformation, dissipation and breakups
Deformation vs. local Weber

\[ \log_{10}(S^*) = We^{0.066} \]

\[ We = \frac{\rho u_d^2 d}{\sigma} \]
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